A Computer Based Mathematical Formulation of Location Problem in Distributed Service Network

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Abstract
Distributed Service Network is a wide term related to distribution and travelling i.e. distribution of resource among facilities located at various locations and travelling of resource along a distributed network. Policy making problems in distributed service networks can be clearly classified into a number of hierarchical levels. The levels are distinguished by time horizon of the problem, by amount of cost involved in the implementation of a solution, and by the political implications of the solution. In the public sector, it includes ambulance, fire, police and other services. In the private sector courier, taxi, repair, maintenance and the like are considered to be distributed service network.

This paper considers a location problem in Distributed Service Network. The location problem aims to determining the number, the position and the mission of required distribution centers with in the populated region. The main objective to minimizes the total transportation duration from the distribution centers to demand points. The result obtained in our experimental study show that the computing time required by the proposed method may be large for some instances.
Keywords: *Emergency response, location problems, p – median.*

INTRODUCTION

Location problems may be classified in several ways such as the optimality criteria,the types of network, single or multifacility location and degree of generality in the models.The most important issue raised in the process of solving location problems is the selection of suitable optimality criterion or objective function. In the ambulance problem, a possible criterion is to minimize the average distance an ambulance must travel in order to reach a random incident or the distance a patient must travel to reach the closest emergency medical service station. Since response time is one of the more important considerations in the design of an emergency service system, a more suitable criterion may be to minimize travel times rather than travel distances.

This paper focuses on the logistics aspect of the response phase and more precisely on Location problems. The location problem aims at designing a network for distributed services. It mainly consists in determining the number, the position and the mission of required location within affected region. The transportation problem deals with the distribution of objects from distribution centres to demand point. When both problems are solved simultaneously, we speak about a location problem.

In any location problem, points of demand must be identified as the demands can occur anywhere on the network or at discrete points i.e. nodes. Both are important in minimax location problems. In general, median problem deals with identifying locations for stationary facilities such that the average distance from a node to the nearest facility would be minimized. This paper deals with a objective location problem in sudden and unforeseen accidents.. The main objective is to minimize the total transportation duration of needed products from the distribution centres to the demand points.

Mathematical Formulation:

Let us suppose that there are nine nodes in a network and we want to find suitable optimality criterion or median zone. We know the demand of each node and the distance between the nodes in units of time. We make zones by including nodes under the condition of constraints of demand and the largest shortest distance.

Suppose a number of stationary facilities, say p, are to be located on a network. To solve this problem, we have to make one additional assumption : a request for service will always be served by the closed facility. Under this assumption, we like to find a set of points on network such that the average shortest distance any node of the network and the closest facility would be minimized.
Based on Hakimi's theorem, we consider that the search for an optimal set of p locations may be confined only to nodes of the network. Hence, one way to solve the problem is simply by enumerating all the possible subsets of p nodes and calculating the shortest distance between any nodes and its closest facility.

Let us consider an undirected network G (N, L), where N is the set of nodes and L is the set of links. The objective is to locate p facilities on the network (p≥ 1) . Two main assumptions for the problem are:

(i) The link travel times are deterministic,

(ii) The closest facility to any possible call for service always contains at least one service unit that is free to be dispatched for service.

For a single facility the one median problem is to find a point x on the network such that

$$\sum_{j=1}^{n} h_j \; d(x,j) \quad \text{for any x in G (N,L)………(1)}$$

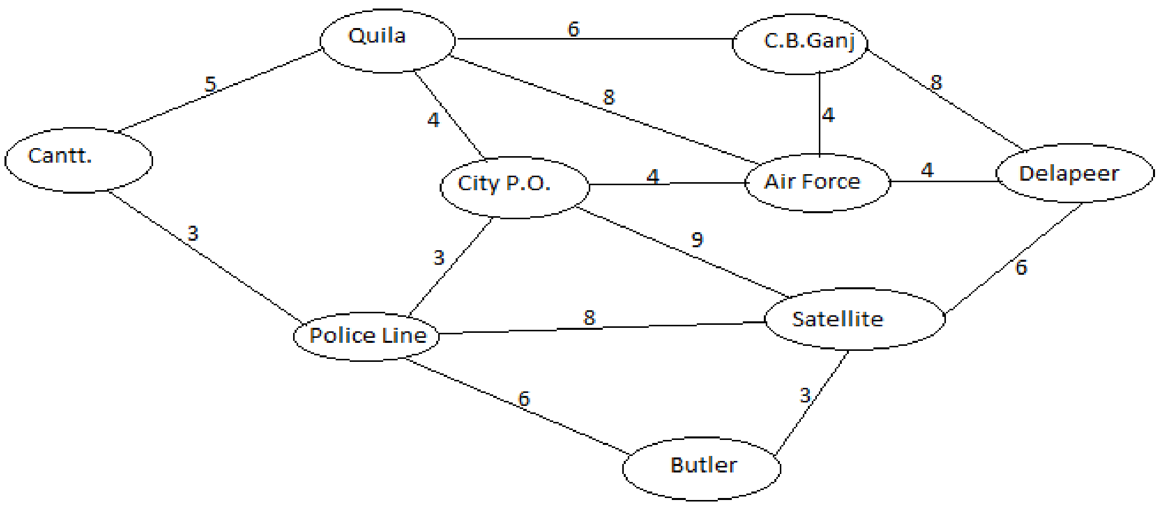
Where h_j is the proportion of the demand that is generated at node j. ($\sum_{j=1}^{n} h_j = 1$) and
d (x, j) is the shortest distance between the points x and j. Equation (1) ensures that x*, the optimal location, minimizes the expected travel time to random call. For a general number of facilities, we denotes by x_p a set of p points x_p = $(x_1 , x_2 \ldots \ldots \ldots . x_p)$. To find the optimal set of locations x_p*, the p-median guarantees that

$$\sum_{j=1}^{n} h_j \; d(x_p *, j) \; \leq \; \sum_{j=1}^{n} h_j \; d(x_p, j) \qquad \text{For } x_p \text{ in G (N,L)……..(2)}$$

Where d (x_p, j) = min [d(x_p, j)].

According to the theorem, "There exists at least one optimal solution set of locations x_p* on the nodes of the network".
The above theorem concludes that the search for the optimal set of locations can be restricted to a finite set of nodes instead of an infinite set of all the points in G (N,L).

A PICTORIAL REPRESENTATION:
Let us consider nine areas in Bareilly City and we want to find the optimal location into these nine areas:



Distances D(x,j) → **Demand (h_j) ↓**	**Nodes**	**Cantt**	**Police line**	**City post office**	**Satellite**	**Butler**	**Quila**	**C.B. Ganj**	**Air force**	**Delapeer**
0.14	Cantt.	0	3	6	11	9	5	11	10	14
0.10	Police line	3	0	3	8	6	7	11	7	11
0.115	City post office	6	3	0	9	9	4	8	4	8
0.135	Satellite	11	8	9	0	3	13	14	10	6
0.125	Butler	9	6	9	3	0	13	17	13	9
0.13	Quila	5	7	4	13	13	0	6	8	12
0.06	C.B. Ganj	11	11	8	14	17	6	0	4	8
0.07	Air force	10	7	4	10	13	8	4	0	4
0.125	Delapeer	14	11	8	6	9	12	8	4	0

In the above table , calculate the distances between areas choose one by one. In first row, write the distances from cantt area to all nine areas. In second row, write the distances from police line and so on. The D^r matrix for the network is :

Demand	**Nodes (v_i)**	**Cantt**	**Police line**	**City post office**	**Satellite**	**Butler**	**Quila**	**C.B.Ganj**	**Air force**	**Delapeer**
0.14	Cantt.	0	0.42	0.84	1.54	1.26	0.70	1.54	1.40	1.96
0.10	Police line	0.30	0	0.30	0.80	0.60	0.70	1.10	0.70	1.10
0.115	City post office	0.69	0.35	0	1.04	1.04	0.46	0.92	0.46	0.92
0.135	Satellite	1.49	1.08	1.22	0	0.41	1.76	1.89	1.35	0.81
0.125	Butler	1.13	0.75	1.13	0.38	0	1.63	2.13	1.63	1.13
0.13	Quila	0.65	0.91	0.52	1.69	1.69	0	0.78	1.04	1.56
0.06	C.B.Ganj	0.66	0.66	0.48	0.84	1.02	0.36	0	0.24	0.48
0.07	Air force	0.70	0.49	0.28	0.70	0.91	0.56	0.28	0	0.28
0.125	Delapeer	1.75	1.38	1.00	0.75	1.13	1.5	1.00	0.50	0
Sum	**J(x) =**	**7.37**	**6.04**	**5.77**	**7.74**	**8.06**	**7.67**	**9.64**	**7.32**	**8.24**

If we do the sum of all the columns, we observe that column III (City post office) has the minimum sum . Hence a median of the network is v_i with an expected travel time of $J(v_3)$ = 5.77 units.

Program to implement above calculation:

```
clear all
clc
A=[0 3 6 11 9 5 11 10 14;3 0 3 8 6 7 11 7 11;6 3 0 9 9 4 8 4 8;11 8 9 0 3 13 14 10 6;9 6 9 3 0 13 17 13 9;5 7 4 13 13 0 6 8 12;11 11 8 14 17 6 0 4 8;10 7 4 10 13 8 4 0 4;14 11 8 6 9 12 8 4 0];
B=[0.14 0.10 0.115 0.135 0.125 0.13 0.06 0.07 0.125];
Distances=A
Demand=B'
for i=1:9
  A(i,:)=A(i,:).*B;
end
A=A';
for i=1:9
  C(i)=sum(A(:,i));
end
Absolute_Median=min(C)
```

Conclusion :

In the public sector like ambulance, fire, police and other services and in private sector like courier, taxi, repair, maintenance etc. When we start a service policy, our first aim is to partition a network into smaller networks. The location problem aims to determining the number, the position and the mission of required distribution centres within the populated region. We have nine nodes in Bareilly city and we had to find a median node (**City post office**) for ambulance services where we have to locate a facility. We minimize the expected travel time to random call. We can apply this model to fire, police and other services.

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