Modified GM (1, 1) Models for Demand Forecasting of wheat in Pakistan

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Abstract
In this paper, we used Grey modeling as a tool to forecast the demand of wheat in Pakistan. Forecasting of demand with high accuracy is also very important for industrial production. The main objective of the study is to analyze advantages of grey systems forecasting models. For this purpose two new modified forms of GM (1, 1) model are proposed. The original and modified model test will be used to forecast the future demand. Through simulated results, this study showed that both of two modified models are suitable but first modified GM (1, 1) is excellent model in forecast with less average relative error. Hence first modified GM (1, 1) model is strongly suggested for forecast the demand wheat production in Pakistan.
Keywords: *Grey modeling; Demand Forecasting; GM (1, 1) Model; Grey differential equation; Relative Error; Simulated values.*

1. Introduction

The aim of production, planning and control processes is to optimize the combination of production factors through proper planning, scheduling and controlling issues. During this period the main trigger function is demand forecasting with high accuracy, although in current Era the demand forecasting has still kept its own importance in Research area for production system in competition conditions. The goal of grey system and its application is like a bridge between other Sciences, thus we can say that the Grey system theory is now useful for variety of specialized fields.

In 1982 professor Deng first introduced grey system theory and then with the passage of time it has been used successfully in many fields. The main model of grey theory for prediction is GM (1, 1). Sifeng Liu (2009) described the reason for study that how grey system theory appeared into world of knowledge, the surprising evolution that has made the world of knowledge in Theory of grey systems and its comprehensive presentations in the whole field of knowledge. He made a comparison of grey uncertainty among other kinds of insecurity such as stochastic randomness, fuzzy and rough randomness. He conducted an inventive of grey system model and successfully introduced independently.

Wang Jian and QI Xiaoli (2010) considered that based on data analysis in China, the study showed that new inventive talents in higher studies are directly related to industrial structure. The enrichment of human capital and increase in the number of students has an imperative effect on industrial development. Yi Zhang (2012) improved grey Verhulst model. He analyzed the inaccuracy of grey Verhulst model. He improved the precision by using the proposed model. He derived the improved equation based upon the properties of grey verhuslt model.

Erdal Aydemir et al., (2013) compared the GM (1, 1) model with ARIMA (1, 1).They further used grey trigonometric forecasting model for demand forecasting of copper wire. They proved that trigonometric grey forecasting model is the best model as it showed the best MAPE value. Yingjie Yang and Sifeng Liu (2013) presented latest version of geometry by combining traditional geometry with grey system. In grey geometry the boundaries of shapes were defined by a grey scope slightly than a clear boundary. In a series of acquaints the outer and inner limit boundaries and thus brings differences among traditional, geometry and the proposed one.

Guangshu Xu *et al*., (2015) studied the problem related to agriculture product logistics in community. They utilized Grey model for demand forecasting of agriculture products. This study helped in solving the supply and demand issues of community shops and was also helpeful in satisfying consumer's demand that maximized the benefits of operators.

Zhen Yun Hu *et al.*, (2015) proved that the improved prediction series grey neural network has higher precision and it is a strong prediction method .They predicted the Industrial water demand. They compared grey neural network model with traditional and grey neural models. After checking the feasibility of models they proved that grey model has higher precision as compared to other models. Wang (2016) used an improved grey multivariate model for forecasting energy consumption in China. He proved that the optimized grey multivariate model gives greater accuracy than GM (1, n) and GM (1, 1). To increase the forecasting precision of GMC (1, n) the authors of this paper have added *n* interpolation coefficient. The modeling and forecasting results show that the grey multivariate model promotes the forecasting precision.

Forecasting is a very useful technique in agriculture. Due to rapid increase in population demand forecasting is a big problem in many countries. Some countries are facing the problem of lack of information and small data to overcome these problems. A technique is introduced in 1982 named as grey modeling. It is a very useful technique with lack of information.

Though lots of research has been done in the field of model fitting, but there are still some difficulties in construction of grey models. In this paper we propose different modifications in GM (1, 1) model and results are compared with original GM (1, 1) model. Modification is made in GM (1, 1) model for forecasting wheat demand in Pakistan. For this purpose we used secondary data and then results are compared with existing GM (1, 1) Model. In second part we explained the existing GM (1, 1) model and after this in third part we modified GM (1, 1) model and then in fourth and fifth part we apply the data and model accuracy test.

2. GM (1, 1) model

The origin of grey theory was started in 1982 by "Professor Deng". Different time series models can be used for forecasting but Deng (1989) proved that for small data Grey models gives improved results as compared to other models. The construction of Grey Models is basically related to Time series model. The GM (1, 1) is first order one variable model and the derivation of the model is given below:

The Grey differential equation is

$$\frac{dy}{dt} + ay = b \tag{1}$$

The original values

$x^{(0)} = x^{(0)}(1),\ x^{(0)}(2), \dots \dots \dots x^{(0)}(n)$

be a positive sequence, where "$x^{(0)}(k) \geq 0$ " and k = 1, 2, ..., n

Accumulating Generating Operation sequence of $x^{(0)}$ with

$$x^{(1)} = x^{(1)}(1), x^{(1)}(2), \dots \dots \dots x^{(1)}(n) \tag{2}$$

where

$x^{(1)}(k) = \sum x^{(1)}(i)$

k = 1, 2, ….,n and $Z^{(1)}$ is the mean generated sequence of $x^{(1)}$ given by

$Z^{(1)} = Z^{(1)}(1),\ Z^{(1)}(2) \dots . Z^{(1)}(n)$

$Z^{(1)}(k) = 0.5Z^{(1)}(k),\ Z^{(1)}(2) \dots . Z^{(1)}(k-1)$

$\hat{a} = (a, b)^T$

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ . \\ . \\ . \\ x^{(0)}(n) \end{bmatrix} \quad , \quad B = \begin{bmatrix} -Z^{(0)}(2) & 1 \\ -Z^{(0)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -Z^{(0)}(n) & 1 \end{bmatrix}$$

$$a = \frac{\frac{1}{n-1} \sum_{k=2}^{n} x^{(0)}(k). \sum_{k=2}^{n} Z^{(1)}(k) - \frac{1}{n-1} \sum_{k=2}^{n} x^{(0)}(k).z^{(1)}(k)}{\sum_{i=2}^{k} [z^{(1)}(k)]^2 - [\sum_{i=2}^{k} z^{(1)}(k)]^2}$$

$$b = \frac{1}{n-1} \left[\sum_{k=2}^{n} x^{(0)}(k) + a \sum_{k=2}^{n} z^{(1)}(k)\right].$$

In that case least squares estimates of the "Grey differential equation" the GM (1, 1) model is

$x^{(0)}(k) + a\, z^{(1)}(k) = b$ (3)

Satisfies

$\hat{a} = (B^T B)^{-1} B^T Y$

3. Modification in GM (1, 1) Model

3.1 First Modification in GM (1, 1) Model

In many practical applications, many researchers found that the GM (1, 1) model is appropriate for gradually increasing data but the fitting outcome with rapidly increasing data was unacceptable. Therefore many scholars have made improvements in GM (1, 1) model. Quin Wan (2010) improved prediction of GM(1,1) model based on ratio modeling method but drawback of the method is that it does not work with solving inverse matrix so still there needs an improvement, so for the modification in GM (1, 1) Model we transformed the demand data by using the following transformation.

$u = x^{(0)}(k)\, e^{-ak}$ (4)

1) The original values are represented by

$$u^{(0)} = u^{(0)}(1), u^{(0)}(2), \dots \dots \dots u^{(0)}(n)$$

2) Then the 1-IAGO sequence $U^{(0)}$ of $U^{(1)}$ is represented by

$$u^{(1)} = u^{(1)}(1), u^{(1)}(2), \dots \dots \dots u^{(1)}(n)$$

$$u^{(1)}(k) = \sum_{i=1}^{k} u^{(0)}(i) \quad , k = 1,2, \dots . n$$

The sequence of $Z^{(1)}$ is obtain by following formula

$$z^{(1)}(k) = \frac{1}{2}[z^{(1)}(k) + z^{(1)}(k-1)]$$

$$Z^{(1)}(k) = 0.5u^{(1)}(k) + 0.5\, u^{(1)}(k-1)$$

3) The FMGM (1, 1) Model is:

$u^{(0)}(k) + cz^{(1)}(k) = d$ (5)

4) The differential equation of Verhulst is

$$\frac{dy}{du} + cy = d$$

5) Calculate the parameters using least square method.

$\hat{a} = [B^T B]^{-1} B^T Y$

$$Y = \begin{bmatrix} u^{(0)}(2) \\ u^{(0)}(3) \\ .. \\ .. \\ u^{(0)}(n) \end{bmatrix} \quad , \quad B = \begin{bmatrix} -Z^{(0)}(2) & 1 \\ -Z^{(0)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -Z^{(0)}(n) & 1 \end{bmatrix}$$

$$c = \frac{\frac{1}{n-1} \sum_{k=2}^{n} u^{(0)}(k).\sum_{k=2}^{n} Z^{(1)}(k) - \frac{1}{n-1} \sum_{k=2}^{n} u^{(0)}(k).z^{(1)}(k)}{\sum_{i=2}^{k} [z^{(1)}(k)]^2 - [\sum_{i=2}^{k} z^{(1)}(k)]^2}$$

$$d = \frac{1}{n-1} \left[\sum_{k=2}^{n} u^{(0)}(k) + c \sum_{k=2}^{n} z^{(1)}(k)\right]$$

6) Change the variable "u" by using following relation

$$x^{(0)}(k) = u\boldsymbol{e^{ak}} \qquad (6)$$

3.2 Second Modification in GM (1, 1) Model

Zhen Yun and Hu (2015) used method of improved series using neural network but still they are applicable only in slowly increasing data so we used modification in the model by changing its parameters and then transforming the data in the following relation

$$v = x^{(0)}(\mathrm{k}) + \mathrm{B}e^{-Ak} \qquad (7)$$

taking $\mathrm{B} = \frac{2b}{2+a}$ and $\mathrm{A} = ln\frac{2-a}{2+a}$

where $$a = \frac{\frac{1}{n-1} \sum_{k=2}^{n} x^{(0)}(k).\sum_{k=2}^{n} Z^{(1)}(k) - \frac{1}{n-1} \sum_{k=2}^{n} x^{(0)}(k).z^{(1)}(k)}{\sum_{i=2}^{k} [z^{(1)}(k)]^2 - [\sum_{i=2}^{k} z^{(1)}(k)]^2}$$

$$b = \frac{1}{n-1} \left[\sum_{k=2}^{n} x^{(0)}(k) + a \sum_{k=2}^{n} z^{(1)}(k)\right]$$

1) The original values are represented by

$$\mathrm{v}^{(0)} = \mathrm{v}^{(0)}(1), \mathrm{v}^{(0)}(2), \dots \dots \dots \mathrm{v}^{(0)}(\mathrm{n})$$

2) Then the 1-IAGO sequence $v^{(0)}$ of $v^{(1)}$ is represented by

$$v^{(1)} = v^{(1)}(1), v^{(1)}(2), \dots \dots \dots v^{(1)}(n)$$

$$v^{(1)}(k) = \sum_{\mathrm{i}=1}^{\mathrm{k}} \mathrm{v}^{(0)}(\mathrm{i}) \quad , k = 1,2, \ldots . n$$

and the arrangement of $Z^{(1)}$ is obtain by following formula

$$z^{(1)}(k) = \frac{1}{2}[z^{(1)}(k) + z^{(1)}(k-1)]$$

$$z^{(1)}(k) = 0.5\mathrm{v}(k) + 0.5\mathrm{v}^{(1)}(k-1)]$$

Assume that

$$\hat{a} = (a,b)^T$$

is a sequence of parameters and

$$Z^{(1)} = z^{(1)}(1), z^{(1)}(2), \ldots \ldots \ldots . z^{(1)}(n).$$

3) The SMGM (1, 1) Model is:

$$v^{(0)}(\mathrm{k}) + \mathrm{a}\, z^{(1)}(\mathrm{k}) = \mathrm{b} \qquad (8)$$

4) The differential equation is

$$\frac{\mathrm{dy}}{\mathrm{dt}} + \mathrm{ay} = \mathrm{b}$$

5) Calculate the parameters using least square method.

$$\hat{a} = [B^T B]^{-1} B^T Y$$

$$Y = \begin{bmatrix} v^{(0)}\,(2) \\ v^{(0)}\,(3) \\ . \\ . \\ . \\ v^{(0)}\,(n) \end{bmatrix} \quad , \quad B = \begin{bmatrix} -Z^{(0)}\,(2) & 1 \\ -Z^{(0)}\,(3) & 1 \\ . & . \\ . & . \\ . & . \\ -Z^{(0)}\,(n) & 1 \end{bmatrix}$$

6) Change of variable "v" by using following relation

$$x^{(0)}(k) = v - \mathrm{B}\,\mathrm{e}^{-\hat{\mathrm{a}}\mathrm{k}} \qquad (9)$$

4. Data Accuracy

To check the accuracy of data, we used Quasi smoothness equation

$$\rho(k) = \frac{x^0(k)}{x^1(k-1)}$$

with $\rho(\mathrm{k}) < 0.5$ and $\mathrm{k} > 3$.

For Law Quasi Exponentially, we have

$$\sigma^{(1)}(k) = \frac{x^1(k)}{x^1(k-1)}$$

$$\sigma^{(1)}(k) \in [1, 1.5]$$

Table 1: Quasi smoothness and law of Quasi exponentially for wheat demand

K	$\rho(k)$	$\sigma^{(1)}(k)$
4	0.2601	1.2601
5	0.2081	1.2081
6	0.1751	1.1751
7	0.1496	1.1496

After calculating the values of ρ(k) and $\sigma^{(1)}$ (k) it can be observed from Table 1 that the values lies within a required range.

$\rho(k) < 0.5$

$\sigma^{(1)}(k) \in [1,1.5]$.

Hence both requirements satisfied.

5. Model accuracy

To check the accuracy of the GM (1, 1) model we calculate mean square error ratio and minimum error probability. We have

$$e^{(o)} = \{e(1),\ e(2), \dots\dots\dots..,\ e(n)\}$$

$$e^{(o)} = \{x^{(o)}(1) - \hat{x}^{(0)}(1),\ x^{(o)}(2) - \hat{x}^{(0)}(2),\ \dots\dots\dots..,\ x^{(o)}(n) - \hat{x}^{(0)}(n)\}$$

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x^{(o)}(k)$$

$$S_1{}^2 = \frac{1}{n}\sum_{k=1}^{n}\left(x^{(0)}(k) - \bar{x}(k)\right)^2$$

which is the mean and variance of $x^{(o)}$ respectively. Furthermore

$$\bar{e} = \frac{1}{n}\sum_{k=1}^{n} e(k)$$

$$S_2{}^2 = \frac{1}{n}\sum_{k=1}^{n}(e(k) - \bar{e}(k))^2$$

are the mean and variance of error respectively.
Mean square error ratio is denoted by C

$$C = \frac{S_2}{S_1}$$

P represents Minimum error of probability

$$P = \{|e(k) - e|\} \leq 0.6745 S_1$$

Table 2: Recommended values for assessment of precision level

Precision grade	**C**	**P**
Superb	≤ 0.35	$\geq 95\%$
Fine	≤ 0.50	$\geq 80\%$
Agree	≤ 0.65	$\geq 70\%$
Disagree	≥ 0.80	$\leq 60\%$

Table 3: Results required for validity of model for wheat demand using first modified form

K	Residual error	$\lvert e(i) - \bar{e} \rvert$
1	0	1.3828
2	193.0419	191.6590
3	-111.3120	112.6049
4	-319.9084	321.2913
5	166.8768	165.4939
6	50.1754	48.7925
7	128.8743	127.4914
8	-96.6849	98.0678

1) Posterior error ratio test

$\bar{x}$ = 23625
$\bar{e}$ = 1.38289
s_1 = 685.1095
s_2 = 161.3628

After calculating Mean square error ratio, we can observe
C = 0.2355 < 0.35
This shows the superb precision level.

2) Small error Probability test
P ($|e(i) - \bar{e}| < 462.1063$) = 1 > 0.95
So test satisfy at level 1.

Table 4: Results required for validity of model for wheat demand using second modified form

K	Residual error	$\|e(i) - \bar{e}\|$
1	0	14.4904
2	191.1730	176.6826
3	-113.0687	127.6826
4	-321.3685	335.8589
5	166.2192	151.7288
6	49.6398	35.1494
7	128.8377	114.3473
8	-96.2434	110.7338

1) Posterior error ratio test

$$\bar{x} = 23625$$
$$\bar{e} = 14.4904$$
$$s_1 = 685.1095$$
$$s_2 = 168.11205$$

$C = 0.24 < 0.35$

2) Small error Probability test

$P\ (\ |e\ (i) - \bar{e}\ | < 462.1063\ ) = 1 > 0.95$

so test satisfies at level 1.

Table 5: Results of wheat demand using GM (1, 1) model

Years	Actual Values Wheat Demand	FMGM(1, 1) Simulated values	SMGM(1, 1) Simulated values	Original GM(1, 1) Simulated values
2008	22800	22800	22800	22800
2009	23000	22807	22809	22809
2010	23000	23111	23113	23113
2011	23100	23420	23421	23421
2012	23900	23733	23734	23734
2013	24100	24050	24050	24050
2014	24500	24371	24371	24371
2015	24600	24697	24696	24697

The results show that the simulated values of modified models are comparatively closer with actual values than original GM (1, 1) model. According to results listed in Table 5 simulated

Values of FMGM (1, 1) model are much closer with actual values as compared to SMGM(1, 1) and original GM (1, 1).

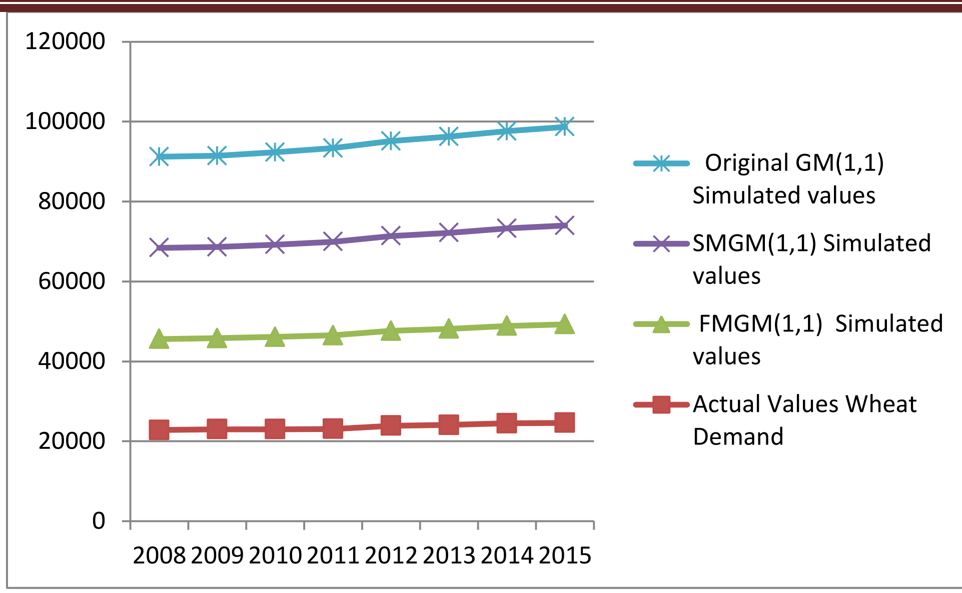

Figure 1: Graphical representation of wheat demand simulated values using grey models

From above graph we conclude that forecasted values calculated by modified GM (1, 1) are more precise as compared to original GM (1, 1).

Table 6: Comparison between Relative Errors of Wheat Demand

Years	Relative Errors (%)		
	FMGM(1, 1)	SMGM(1, 1)	Original GM(1, 1)
2008	0.0000	0.0000	0.0000
2009	0.8393	0.8312	0.8297
2010	0.4840	0.4916	0.4931
2011	1.3849	1.3912	1.3927
2012	0.6982	0.6955	0.6940
2013	0.2082	0.2060	0.2044
2014	0.5260	0.5259	0.5243
2015	0.3930	0.3912	0.3928
Average	0.5667	0.6323	0.6473

It can be observed from Table 6 that the forecasting accuracy has been further improved with modified GM (1, 1) models. The average relative error of predictive data is reduced from 0.6473% to 0.5667% using First modified GM (1, 1) model or FMGM (1, 1) model and 0.6473% to 0.6323% using Second modified GM(1, 1) model or SMGM (1, 1). So we can say that the modified GM (1, 1) models.

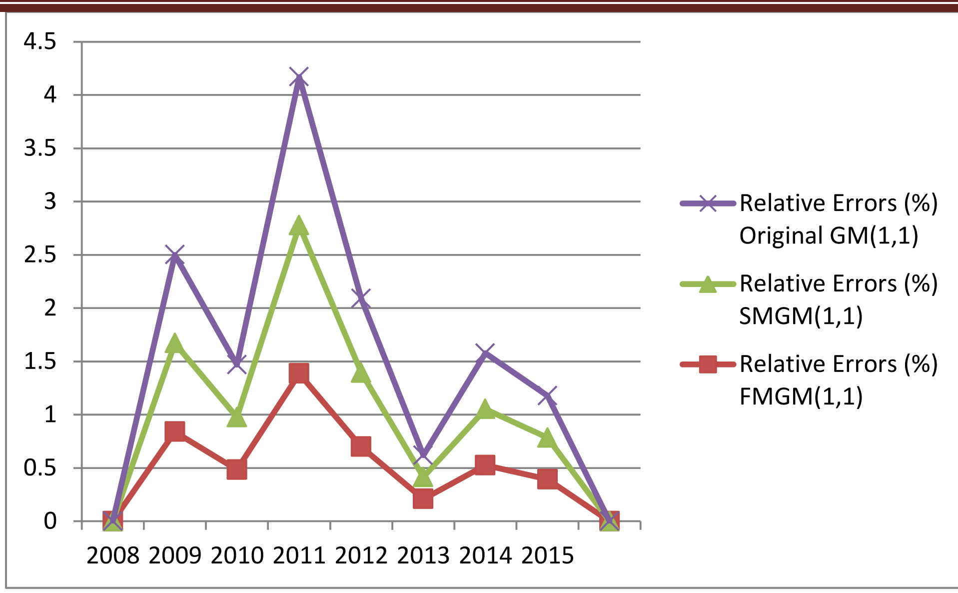

Figure 2: Graphical comparisons between relative errors of wheat demand using original GM (1, 1) FMGM (1, 1) and SMGM (1, 1).

From above graph it can be seen that relative errors calculated by Modified GM (1, 1) are smaller as compared to original GM (1, 1). The best fitted model is one which gives small residual errors or relative errors.

Table 7: Comparison between Forecasted values of Wheat Demand

Years	FMGM(1, 1)	SMGM(1, 1)	Original GM(1, 1)
2016	25027	25026	25026
2017	25361	25359	25360
2018	25701	25698	25698
2019	26043	26042	26041
2020	26391	26388	26388
2021	26743	26740	26740
2022	27100	27097	27097
2023	27461	27458	27458
2024	27829	27825	27825
2025	28201	28195	28196

In order to use the modified GM (1, 1) model for long term forecasting, it is necessary to compare the forecasted values of modified and original GM (1, 1) model for next ten years. Table 7 indicates the forecasting comparison between grey models.

6. Conclusions

The demand of wheat has been forecasted with the help of GM (1, 1) models. The results are compared with existing model. The Modifications have been done in GM (1, 1) model and then results are obtained. To check the Reliability of modified models, results are compared with existing GM (1, 1) model. It has been observed that the modified GM (1, 1) models have comparatively less average relative error than original GM (1, 1) model. This study showed that use of modification in the models provided improved results.

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